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**BEHAVIOR OF STIFFENED PLATES. VOLUME II.
OPTIMUM DESIGN**

Charles O. Heller

**United States Naval Academy
Annapolis, Maryland**

June 1968

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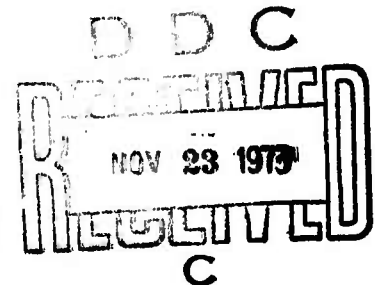
UNITED STATES NAVAL ACADEMY
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Engineering Department

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Behavior of Stiffened Plates,
Volume II: Optimum Design

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ABSTRACT

An optimization study of various configurations of orthogonally stiffened plates is carried out and the relative efficiencies, from the standpoint of minimum weight, are generated for sandwich, integrally stiffened, and externally stiffened rectangular plates under compressive loads.

An attempt is made, from the minimal amount of data available, to introduce minimum cost as a parameter for optimum design.

DESCRIPTORS

Integral stiffening

Optimization

Orthotropy

Plates

Rib-stiffening

Sandwich

Structural analysis

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INTRODUCTION

This report is a sequel to an analytical study of the structural behavior of stiffened plates (Reference 7) carried out at the U. S. Naval Academy, with the support of the Naval Academy Research Council.

In this report, an optimization study, based upon minimum weight, is performed for various configurations of orthogonally stiffened plates under uniaxial compressive loading. The technique used throughout the optimization study is that of Gerard (Reference 6)--based upon the criteria that buckling signifies failure and that all buckling-mode failures occur simultaneously.

The major contributions of the work are the comparison of relative efficiencies of various sandwich, integrally stiffened, and externally stiffened plates from the standpoint of minimum weight, and the introduction of minimum cost as an optimization parameter.

BACKGROUND

Minimum-weight design--the rational selection of a structure and the material for its construction for a particular design requirement--is of great interest because it guarantees economic and efficient utilization of structural materials. In flight vehicles, the importance of minimum weight is self-evident and it is from this general area that most of the techniques of modern structural design have evolved. To the civil engineer, cost is of prime importance and, since cost is generally very closely related to weight, minimum-weight design should be of prime interest. Additionally, the ship designer must guard closely against the over-design of large hulls and, therefore, the principles of minimum-weight design should be followed.

The pioneer effort in the area of structural optimization was that of Shanley who originally published his well-known text (Ref. 19) in 1952. Since that time, many investigators have devoted their research to this field. The most notable in the area of stiffened plates are Gerard (Ref. 6) and Vinson and Shore (Refs. 24-28). Several authors also demonstrate the feasibility of using programming techniques for structural optimization. Prager (Ref. 13) uses linear programming for weight minimization, while Moses (Ref. 12), Schmit et al (Refs. 14-18), and Brown and Ang (Ref. 4) approach the problem through nonlinear programming methods.

The optimization study performed here is based on the stability criterion. Since in-plane compressive loading is most often the controlling factor in the design of such plates, the results of weight-strength analyses of plates subjected to such loads indicate geometrical proportions which, in many cases, can be considered to be optimum proportions (Ref. 1), particularly in the design of flight vehicle structures.

The technique chosen here is that of Gerard (Ref. 6) because it is relatively simple and because it is a rather straight-forward task to transform its results into a form suitable for comparison of the relative efficiencies of various configurations.

WEIGHT-MINIMIZATION TECHNIQUE

The stability of an orthogonally stiffened plate is influenced by three factors which contribute to the total complexity of the actual instability condition. In its exact form, instability of such a plate is mathematically too cumbersome to be expressed in terms of a convenient weight-efficiency parameter, and several simplifying assumptions must be made. The influencing factors are:

- a. The fact that each stiffener is characterized by a flexural and torsional rigidity.
- b. The fact that the deflected form of the skin-stiffener composite is a hybrid between a sinusoidal and a cubic polynomial curve.
- c. The fact that the stiffener spacing influences the preferred wave form of buckling.

Several simplifications which do not seriously affect the accuracy of results (Ref. 5) are introduced: the complications introduced by factors (b) and (c), above, are evaded by assuming that the stiffeners are uniformly spread across the plate width. The torsional rigidity of a stiffener cannot be distributed in the same manner, but it can be assumed, without greatly affecting the accuracy of most solutions, that the torsional rigidity of a stiffener is negligible.

The optimization process performed here is based upon the assumptions that buckling signifies failure and that all buckling-mode failures occur simultaneously. Thus, for

a plate with stiffeners,

$$\sigma_{Ap} = \sigma_{cr}^o - \sigma_{cr}^l, \quad (1)$$

where

σ_{Ap} = applied compressive stress,

σ_{cr}^o = overall (gross) critical buckling stress,

σ_{cr}^l = local (skin) critical buckling stress.

The basic step consists of combining the above stresses such that an expression for the stress in terms of geometry parameters and loading results. The stresses are combined according to

$$(\sigma_{cr}^l)^A (\sigma_{cr}^o)^B (\sigma_{Ap})^C = \bar{\sigma}^{A+B+C}, \quad (2)$$

where the exponents A, B, C are calculated by dimensional analysis.

Next, the geometry parameter in the σ - expression is either maximized or minimized (whichever is appropriate), and thus the optimum dimension ratio relationships are established.

Finally, a "solidity" function--a measure of the weight or the actual plate in comparison to that of a rectangular solid of the same overall dimensions--is introduced for the purpose of comparison with other configurations.

The entire procedure, originally given by Gerard (Ref. 6), is described more clearly in the following analyses of particular stiffening systems.

INTEGRAL LONGITUDINAL STIFFENERS

A simply supported plate, loaded by compressive loading along the y-edges, and stiffened in the direction of the load is shown in Figures 8.1 and 8.2 and analyzed in this section. The gross critical buckling stress of an orthotropic plate in uniaxial compression is well known (Ref. 7):

$$\sigma_{cr} = \frac{2\pi^2}{b^2 t} \left[\sqrt{D_x D_y} + D_y \right] \quad (3)$$

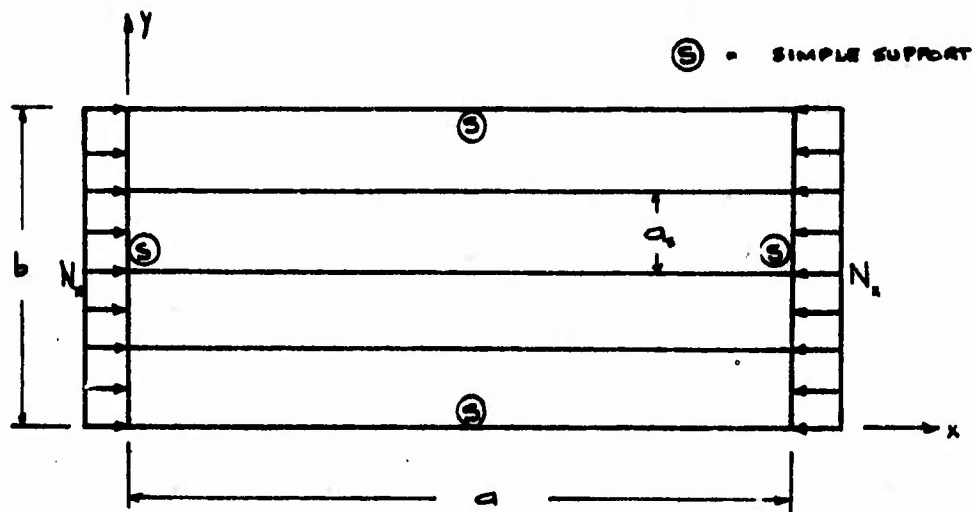


Figure 1

Longitudinally Stiffened Plate

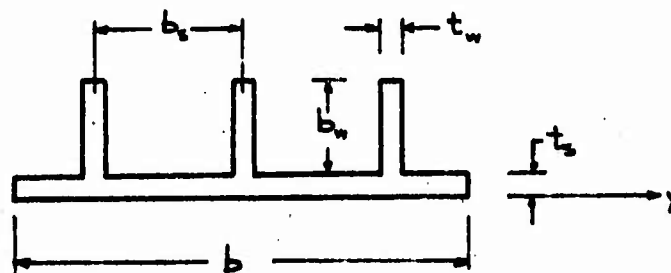


Figure 2

Skin and Stiffener Geometry

If the plate of Figure 2 is replaced by a plate of a constant equivalent thickness (t) and if it is assumed that

$$D_x \gg D_{xy}, \quad D_y \gg D_{xy},$$

and, thus, that

$$D_{xy} \approx 0,$$

the critical buckling stress expression is rewritten:

$$\sigma_{cr}^2 = \frac{2\pi^2}{b_s^2 t} \sqrt{D_x D_y}, \quad (4)$$

where the equivalent thickness,

$$\bar{t} = t_s \left[1 + \frac{b_w t_w}{b_s t_s} \right] \quad (5)$$

Introducing the ratios,

$$r_b = \frac{b_w}{b_s}, \quad r_t = \frac{t_w}{t_s}, \quad (6)$$

the flexural rigidities are given by:

$$\begin{aligned} D_x &= \frac{E}{1-\mu_{xy}\mu_{yx}} \frac{b_w^3 t_s}{12} r_b r_t \left[\frac{4+r_b r_t}{1+r_b r_t} \right], \\ D_y &= \frac{E}{1-\mu_{xy}\mu_{yx}} \frac{t_s^3}{12}. \end{aligned} \quad (7)$$

The critical local buckling stress of the skin between the stiffeners, from elementary theory (Ref. 6),

$$\sigma_{cr}^1 = \frac{k \pi^2 E}{12(1-\mu_{xy}\mu_{yx})} \left(\frac{t_s}{b_s} \right)^2, \quad (8)$$

where k is the buckling coefficient (taken as 4 under the assumption that skin-to-stiffener connection is a hinge).

Eqs. (4) and (8) give the two buckling modes of the stiffened panel which must occur simultaneously in an

optimally designed plate. The applied stress,

$$\sigma_{Ap} = \frac{N_x}{t} \quad (9)$$

Recalling Eq. (1) and (2), the stresses are combined:

$$\begin{aligned} \bar{\sigma}^{A+B+C} &= (\sigma_{cr}^L)^A (\sigma_{cr}^o)^B (\sigma_{Ap})^C \\ &= \left[\frac{k\pi^2 E}{12(1-\mu_{xy}\mu_{yx})} \left(\frac{t_p}{b_s} \right)^3 \right]^A \left[\frac{2\pi^2}{3t} \sqrt{D_s D_p} \right]^B \left[\frac{N_x}{t} \right]^C \end{aligned} \quad (10)$$

For optimum design, the margins of safety must be zero, or

$$\begin{aligned} \sigma_{cr}^o - \sigma_{Ap} &= 0, \\ \sigma_{cr}^L - \sigma_{Ap} &= 0. \end{aligned} \quad (11)$$

After substitution and dimensional analysis (Ref. 6), the exponents become:

$$A = 1, \quad B = 2, \quad C = 4, \quad (12)$$

and

$$\bar{\sigma}^7 = \frac{(2)^2 \pi^4}{(12)^3} \frac{k r_b^3 r_s (4 + r_b r_s)}{1 + r_b r_s} \left[\frac{E}{1 - \mu_{xy} \mu_{yx}} \right]^3 \left[\frac{N_x}{b} \right]^4,$$

or

$$\bar{\sigma} = \alpha \left[\frac{E}{1 - \mu_{xy} \mu_{yx}} \right]^{3/7} \left[\frac{N_x}{b} \right]^{4/7}, \quad (13)$$

where

$$\alpha = \frac{1.1209 [k r_b^3 r_s (4 + r_b r_s)]^{1/7}}{1 + r_b r_s}$$

Since α is expressed as a function of geometry ratios alone, it can be maximized for greatest efficiency. For simultaneous buckling of plate and stiffener,

$$3.978 E \left(\frac{t_s}{b_s} \right)^2 = 0.426 E \left(\frac{t_w}{b_w} \right)^2,$$

from which

$$r_t = 3.056 r_b. \quad (14)$$

From this,

$$\alpha = \frac{1.603 [4r_t^4 + 3.056 r_b^4]^{1/4}}{1 + 3.056 r_b^2}. \quad (15)$$

Maximizing the above,

$$\frac{\partial \alpha}{\partial r_b} = 0, \quad (16)$$

an optimum dimension ratio is computed:

$$r_b^{\text{OPT}} = 0.376. \quad (17)$$

From this,

$$\alpha^{\text{OPT}} = 0.792, \quad r_t^{\text{OPT}} = 1.149. \quad (18)$$

Let

$$S = \frac{\text{WEIGHT OF STIFFENERS}}{\text{WEIGHT OF SKIN}} = \frac{\rho t_w b_w a}{\rho t_s b_s a} = r_t r_b, \quad (19)$$

where ρ is the material density. The optimum weight ratio,

$$S^{\text{OPT}} = r_t^{\text{OPT}} r_b^{\text{OPT}} = 0.432. \quad (20)$$

The optimum stress, from Eqs. (13) and (18),

$$\bar{\sigma}^{\text{OPT}} = 0.792 \left[\frac{E}{1-\mu_s \mu_w} \right]^{3/4} \left[\frac{N_s}{b} \right]^{1/4}. \quad (21)$$

In order to obtain a measure of minimum-weight efficiency, and to be able to compare these efficiencies for various configurations, a "solidity" parameter (Σ) is introduced. This parameter is the ratio of the weight of the actual plate to that of a solid of dimensions $b \times b \times a$:

$$\Sigma = \frac{\rho \bar{t} b a}{\rho b^3 a} = \frac{\bar{t}}{b} \quad (22)$$

Recalling Eqs. (9) and (21),

$$\Sigma^{opt} = \left(\frac{\bar{t}}{b} \right)^{opt} = 1.263 (1 - \mu_{xy} \mu_{yz})^{3/7} \left(\frac{N_x}{bE} \right)^{3/7} \quad (23)$$

This, along with Eq. (5),

$$\bar{t} = t_s (1 + r_s r_t) \rightarrow t_s = 0.698 \bar{t}, \quad (24)$$

is the minimum-weight design expression for a plate with longitudinal, unflanged, integral stiffeners. In the following sections, similar expressions are developed for other configurations and the comparative efficiencies are illustrated by design curves.

INTEGRAL TRANSVERSE STIFFENERS

A simply supported plate, loaded by a compressive loading along the y-edges, and stiffened in a direction normal to the load is shown in Figures 3 and 4 and analyzed in this section. The general procedure is identical to that followed in the preceding section.

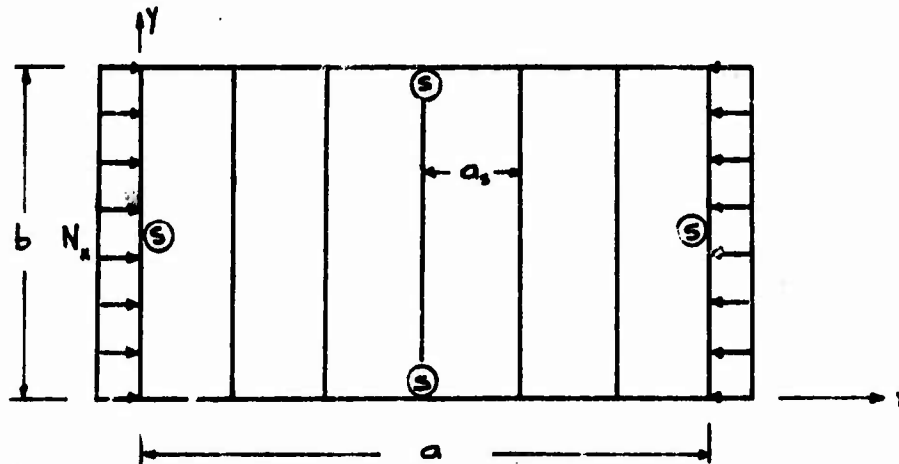


Figure 3

Transversely Stiffened Plate

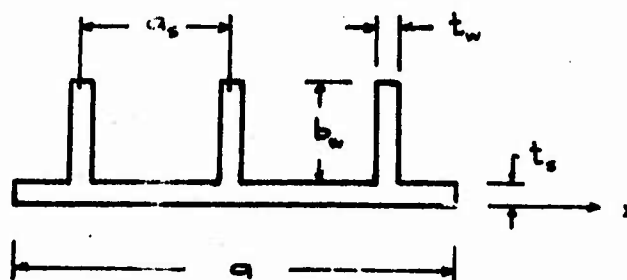


Figure 4

Skin and Stiffener Geometry

In the transversely stiffened plate, the effective thickness from a weight standpoint is \bar{t} , as given by Eq. (5). However, since the stiffeners do not carry the compressive load, the effective thickness from the standpoint of load is simply t_s . Thus, the geometrical ratios, as in Eq. (6),

$$r'_b = \frac{b_w}{a_s}, \quad r'_t = \frac{t_w}{t_s}, \quad (25)$$

and thus the equivalent plate thickness,

$$\bar{t} = t_s \left[1 + \frac{b_w t_w}{a_s t_s} \right] = t_s (1 + r'_b r'_t). \quad (26)$$

The flexural rigidities,

$$D_x = \frac{E}{1 - \mu_{xy} \mu_{yx}} \frac{t_s^3}{12}, \quad (27)$$

$$D_y = \frac{E}{1 - \mu_{xy} \mu_{yx}} \frac{b_w^3 t_s}{12} r'_b r'_t \left[\frac{4 + r'_b r'_t}{1 + r'_b r'_t} \right].$$

The applied and overall critical buckling stresses are expressed by Eqs. (9) and (10). The local buckling stress of a skin panel between stiffeners, however, becomes

$$\sigma_{cr}^L = \frac{\pi^2 D_x}{t_s a_s^2}. \quad (28)$$

For optimum, the three stresses must again be equal, and, from this and an analysis identical to that of the preceding section,

$$\bar{\sigma} = \alpha \left[\frac{E}{1 - \mu_{xy} \mu_{yx}} \right]^{1/2} \left[\frac{N_s}{b} \right]^{1/2}, \quad (29)$$

where α is given in Eq. (13), after the replacement of r_b with r'_b . The solidity, in this case,

$$\begin{aligned}\bar{\Sigma} &= \frac{\bar{t}}{b} = \frac{\bar{t}}{t_s} \left[\frac{N_s}{b\bar{\sigma}} \right] \\ &= \left[\frac{N_s}{b} \frac{1-\mu_{xy}\mu_{yx}}{E} \right]^{3/7} \left\{ \frac{(1+r'_b r_s)^2}{1.209 [r'_b{}^3 r_s (4+r'_b r_s)]^{1/7}} \right\} \\ &= \left[\frac{N_s}{b} \frac{1-\mu_{xy}\mu_{yx}}{E} \right]^{3/7} \beta.\end{aligned}\quad (30)$$

As before, the weight ratio,

$$S = \frac{\rho t_w b_w a}{\rho t_s a_s a} = r'_b r_t, \quad (31)$$

and thus,

$$\beta = \frac{(1+S)^2}{1.209 [S r'_b{}^3 (4+S)]^{1/7}}. \quad (32)$$

For minimum weight, the solidity is minimized with respect to the weight ratio:

$$\frac{\partial \bar{\Sigma}}{\partial S} = 0, \quad (33)$$

resulting in

$$S^{\text{OPT}} = 0.15; \quad r_t^{\text{OPT}} = \frac{0.15}{r'_b}; \quad \beta^{\text{OPT}} = \frac{1.171}{(r'_b)^{2/7}}. \quad (34)$$

Substitution into Eq. (30) yields the minimum-weight design expression for a transversely stiffened plate:

$$\bar{\Sigma}^{\text{OPT}} = \frac{\bar{t}}{b} = \frac{1.171}{(r'_b)^{2/7}} (1-\mu_{xy}\mu_{yx})^{3/7} \left(\frac{N_s}{bE} \right)^{3/7}. \quad (35)$$

INTEGRAL GRID STIFFENING SYSTEM (0° and 90° FROM LOAD)

A simply supported plate with both longitudinal and transverse stiffeners, identical in geometry to those of preceding sections is considered. For a system where all stiffeners are of equal size and with uniform spacing in both directions ($a_s = b_s$), the effective thickness for weight,

$$\bar{t} = t_s (1 + 2r_l r_t), \quad (36)$$

and the effective load-carrying thickness,

$$\bar{t}_l = t_s (1 + r_l r_t). \quad (37)$$

In a configuration such as this, the assumptions of simple supports everywhere (as made by Gerard) can no longer be made because the buckling mode of simply supported skin is not compatible with the buckling mode of simply supported stiffeners (Ref. 9). The skin panels are neither simply supported nor clamped and the buckling coefficient for Eq. (8) is estimated as $k=6$. Using this value and following an analysis identical to those of the preceding sections, the optimum geometrical parameter,

$$\beta^{opt} = 1.420, \quad (38)$$

and the optimum solidity factor,

$$\Sigma^{opt} = \frac{\bar{t}}{b} = 1.420 \sqrt{1 - \mu_x \mu_y} \sqrt{\frac{N_x}{bE}} \quad (39)$$

INTEGRAL GRID STIFFENING SYSTEM (-45° and 45° FROM LOAD)

Gerard (Ref. 6) estimates that the 45-degree grid (Fig. 5) is approximately fifty percent more efficient because the stiffeners participate equally in resisting the compressive load. Using the same reasoning, Eq. (38) is modified:

$$\beta^{opt} = \frac{1.420}{1.5} = 0.946, \quad (40)$$

and the design equation becomes:

$$\sum^{opt} = \frac{\bar{t}}{b} = 0.946 \sqrt{1 - \mu_{xy} \mu_{yx}} \sqrt{\frac{N_x}{bE}} \quad (41)$$

The equivalent thickness,

$$\bar{t} = t_s (1 + 2r_s r_t) \quad (42)$$

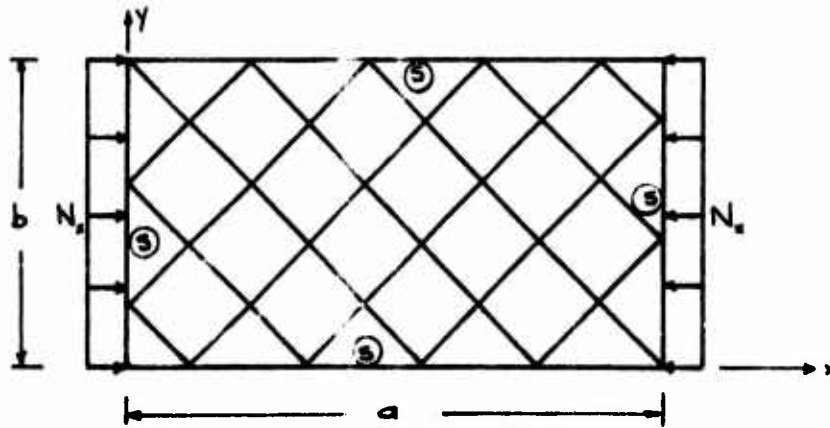


Figure 5

45-Degree Stiffeners

SANDWICH CONSTRUCTION

The sandwich cross-section of Figure 6 is considered for optimization study. Letting ρ_c and ρ_f be the densities of core and facings, respectively, and assuming that their ratio does not deviate greatly from unity, the cross-section can be optimized for minimum weight. Writing the core thickness as a function of the sandwich height, using a constant k :

$$t_1 = kh. \quad (h \gg t_1) \quad (43)$$

Assuming that the facings alone resist tensile and compressive forces due to bending, the moment

$$M_x = 2Pkh\left(\frac{h}{2}\right) = Pkh^2 \quad \rightarrow \quad h = \sqrt{\frac{M_x}{Pk}}. \quad (44)$$

The weight of a unit square of sandwich,

$$W = \rho_c h + 2\rho_f kh = \sqrt{\frac{M_x}{Pk}} (\rho_c + 2\rho_f k). \quad (45)$$

Minimizing the weight for optimum design:

$$\frac{\partial W}{\partial k} = 0, \quad (46)$$

results in

$$k = \frac{\rho_c}{2\rho_f}. \quad (47)$$

Eq. (47) indicates that a sandwich should consist of facings and core of approximately equal weights. This conclusion is

useful as a general guideline and it is utilized in the minimum-weight analyses which follow.

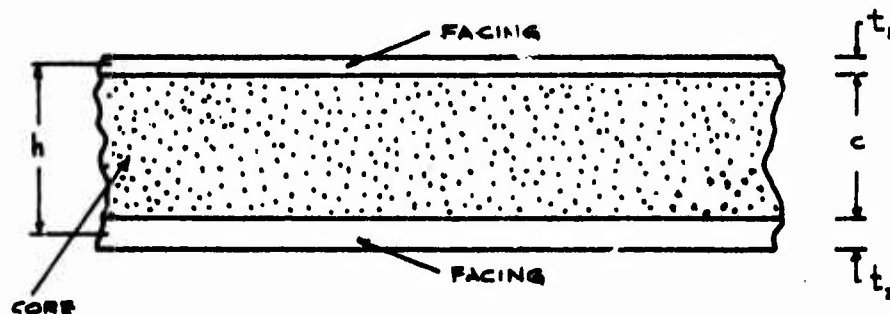


Figure 6
Sandwich Cross-Section

WEB-CORE (BOX-BEAM) SANDWICH

The plate shown in Figure 7 is simply supported and loaded in axial compression (in the direction of the stiffeners). The equivalent thickness is given by:

$$\bar{t} = t_s \left[2 + \frac{\rho_c}{\rho_s + \rho_c} \left(\frac{h}{t_s} - 2 \right) \right], \quad (48)$$

in which the ratio of densities,

$$\frac{\rho_c}{\rho_s + \rho_c} = \frac{(t_s/t_s)}{(l/t_s)} = \frac{t_s}{l} \quad (49)$$

The minimum-weight design expression, from an analysis identical to that used for rib-stiffened plates, is

$$\sum^{OPT} = \frac{\bar{t}}{b} = 1.179 \sqrt{1 - \mu_{xy} \mu_{yx}} \sqrt{\frac{N_x}{bE}} \quad (50)$$

Gerard (Ref. 6) reports that the optimum dimensions of this configuration are given by:

$$\left(\frac{h}{l} \right)^{OPT} = 1.12 \quad (51)$$

and the optimum number of cells is the closest integer to:

$$n^{OPT} = 1.12 \frac{b}{h} \quad (52)$$

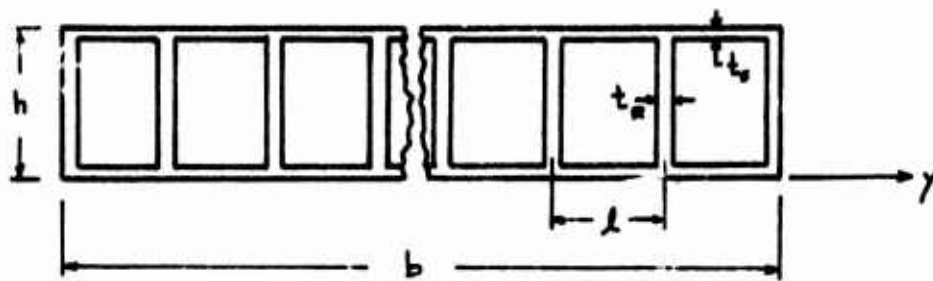


Figure 7
Web-Core Sandwich

TRUSS-CORE SANDWICH

For the truss-core sandwich of Figure 8, the equivalent thickness,

$$\bar{t} = t \left[2 + \frac{\rho_c}{\rho + \rho_s} \left(\frac{h}{t} - 2 \right) \right], \quad (53)$$

in which the ratio of densities,

$$\frac{\rho_c}{\rho + \rho_s} = \frac{(t_c/t)}{(\frac{h}{t} - 1) \cos \theta} \quad (54)$$

The formulas derived by Vinson and Shore (Ref. 27) are converted to the optimum-design form used throughout this report to give:

$$\sum^{OPT} = \frac{\bar{t}}{b} = 0.964 \sqrt{1 - \mu_{xy} \mu_{yx}} \sqrt{\frac{N_x}{bE}} \quad (55)$$

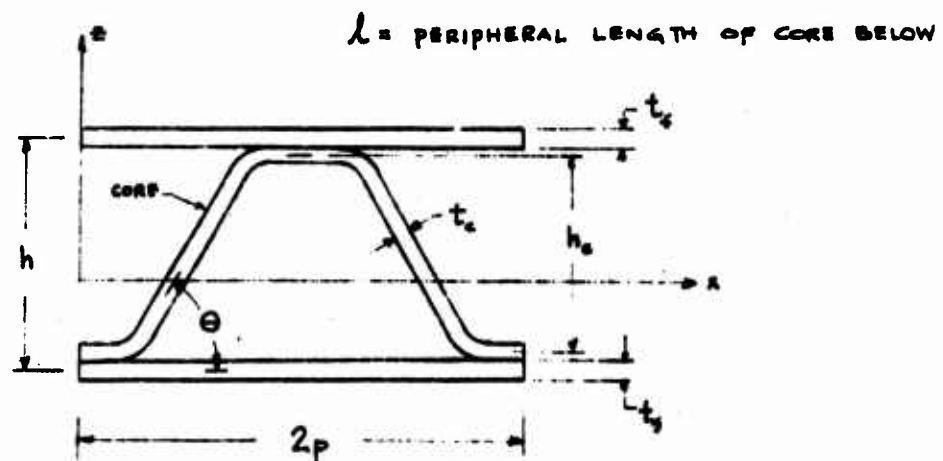


Figure 8
Truss-Core Sandwich

DOUBLE-TRUSS-CORE SANDWICH

A double-truss-core sandwich, simply supported and loaded in axial compression, is considered here and shown in Figure 9.

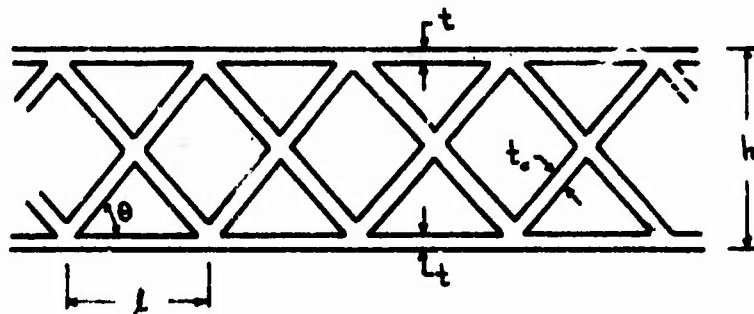


Figure 9

Double-Truss-Core Sandwich

The equivalent thickness is given by:

$$\bar{t} = t \left[2 + \frac{\rho_c}{\rho + \rho_c} \left(\frac{h}{t} - 2 \right) \right], \quad (56)$$

in which the ratio of densities,

$$\frac{\rho_c}{\rho + \rho_c} = \frac{2(t_c/t)}{(\frac{h}{t} - 1) \cos \theta} \quad (57)$$

The minimum-weight design relationship:

$$\sum^{OPT} = \frac{\bar{t}}{b} = 0.911 \sqrt{1 - \mu_1 \mu_2} \sqrt{\frac{N_1}{bE}} \quad (58)$$

HONEYCOMB-CORE SANDWICH

A sandwich plate with honeycomb core, a facing thickness t_f , and total thickness h buckles in three modes, all of which are considered here. The critical buckling stress based on overall stability,

$$\sigma_{cr}^o = \frac{T^2 E}{1 - \mu_{xy} \mu_{yx}} \left(\frac{h}{b} \right)^2. \quad (59)$$

The critical local buckling stress based on face wrinkling,

$$\sigma_{cr}^{lw} = 0.5 E \left(\frac{t_c}{s} \right)^{2/3}. \quad \left(\begin{array}{l} s = \text{CELL WIDTH} \\ t_c = \text{RIBBON THICKNESS} \end{array} \right) \quad (60)$$

The critical local buckling stress based on intracellular wrinkling of the facings,

$$\sigma_{cr}^{li} = \frac{2E}{1 - \mu_{xy} \mu_{yx}} \left(\frac{t_f}{s} \right)^2. \quad (t_f = \text{FACING THICKNESS}) \quad (61)$$

Making use of the conclusion reached earlier (that weights of core and facings should be equal), the equivalent thickness:

$$\bar{t} = 2t_f + \frac{2t_c h}{s}. \quad (62)$$

From an analysis identical to those used previously, the minimum-weight design expression is written:

$$\sum^{opt} = \frac{\bar{t}}{b} = 2.376 (1 - \mu_{xy} \mu_{yx})^{2/3} \left(\frac{N_x}{bE} \right)^{1/3}. \quad (63)$$

UNSTIFFENED PLATE

In order to provide a comparison, the design of a thin, flat, unstiffened, isotropic plate is placed on a basis identical to those of the stiffened plates considered in the preceding paragraphs. Such a plate buckles in one mode, overall buckling, which, for a simply supported plate, is given by:

$$\sigma_{cr}^o = \frac{4\pi^2 E}{12(1-\mu^2)} \left(\frac{t}{b}\right)^2 \quad (64)$$

For optimum, the critical buckling stress equals the applied stress and

$$\bar{\sigma}^{A+B} = (\sigma_{ap})^A (\sigma_{cr}^o)^B = \left(\frac{N_x}{t}\right)^A \left(\frac{\pi^2 E t^2}{12b^2(1-\mu^2)}\right)^B \quad (65)$$

From a dimensional analysis, A=2 and B=1, and

$$\bar{\sigma} = \frac{1.487}{(1-\mu_{xy}\mu_{yx})^{1/2}} \left(\frac{N_x^2 E}{b^2}\right)^{1/3} \quad (66)$$

The solidity,

$$\sum^{opt} = \frac{\bar{t}}{b} = 0.672 (1-\mu_{xy}\mu_{yx})^{1/2} \left(\frac{N_x}{bE}\right)^{1/3} \quad (67)$$

COMPARISONS OF STRUCTURAL EFFICIENCIES

The design expressions given in the preceding paragraphs are plotted here in order to demonstrate the relative efficiencies of various configurations. The Poisson's ratios for all configurations are taken as $\mu_{xy} = \mu_{yx} = 0.3$.

Figure 10 contains plots of the design curves for integrally stiffened plates with unflanged stiffeners and an unstiffened plate. It demonstrates the fact that the plate with 45-degree stiffeners (Figure 5) is the most efficient of this group and that all integrally stiffened plates are better, from the standpoint of minimum weight, than the unstiffened plate capable of carrying the same load.

Figure 11 demonstrates the relative efficiencies of sandwich plates with various core configurations and a flat unstiffened plate. It is shown that the honeycomb-core sandwich is the most efficient of the configurations considered and that all sandwich structures are more efficient than the unstiffened plate.

From Figures 10 and 11, a ranking of the plates, according to their structural efficiencies, is established:

1. Honeycomb-core sandwich
2. Double-truss-core sandwich
3. Single-truss-core sandwich

4. Integrally stiffened plate (45°)
5. Web-core sandwich
6. Integrally stiffened plate (grid)
7. Integrally stiffened plate (longitudinal)
8. Unstiffened plate

Since the above study is based upon stability considerations, the strength limitations of various materials are included in Figures 10 and 11 in the following form:

Aluminum (2014-T6)	-- $(E/\sigma_{cy}) = 178.33$
Fiberglass ("Scotchply")	-- $(E/\sigma_{cy}) = 61.11$
Steel (AISI4130)	-- $(E/\sigma_{cy}) = 200$
Steel (Stainless 301)	-- $(E/\sigma_{cy}) = 433.33$
Titanium (6Al-4V)	-- $(E/\sigma_{cy}) = 126.98$

It is noted that the results presented here are in general agreement with most of the minimum-weight studies carried out by other investigators. There are, however, some exceptions. Anderson (Ref. 1) studies the structural efficiencies of various sandwich structures. Agreement is not reached when his results are transformed into the form presented in this report. Gerard (Ref. 6) assumes that the skin panels in the 0° - 90° and 45° - 45° integral grid plates are simply supported at stiffener intersections. Since this situation appears unrealistic, these edge conditions are considered here to be between the pinned and clamped classifications here, and, thus, the efficiencies of both configurations derived in this report are higher than those of Gerard.

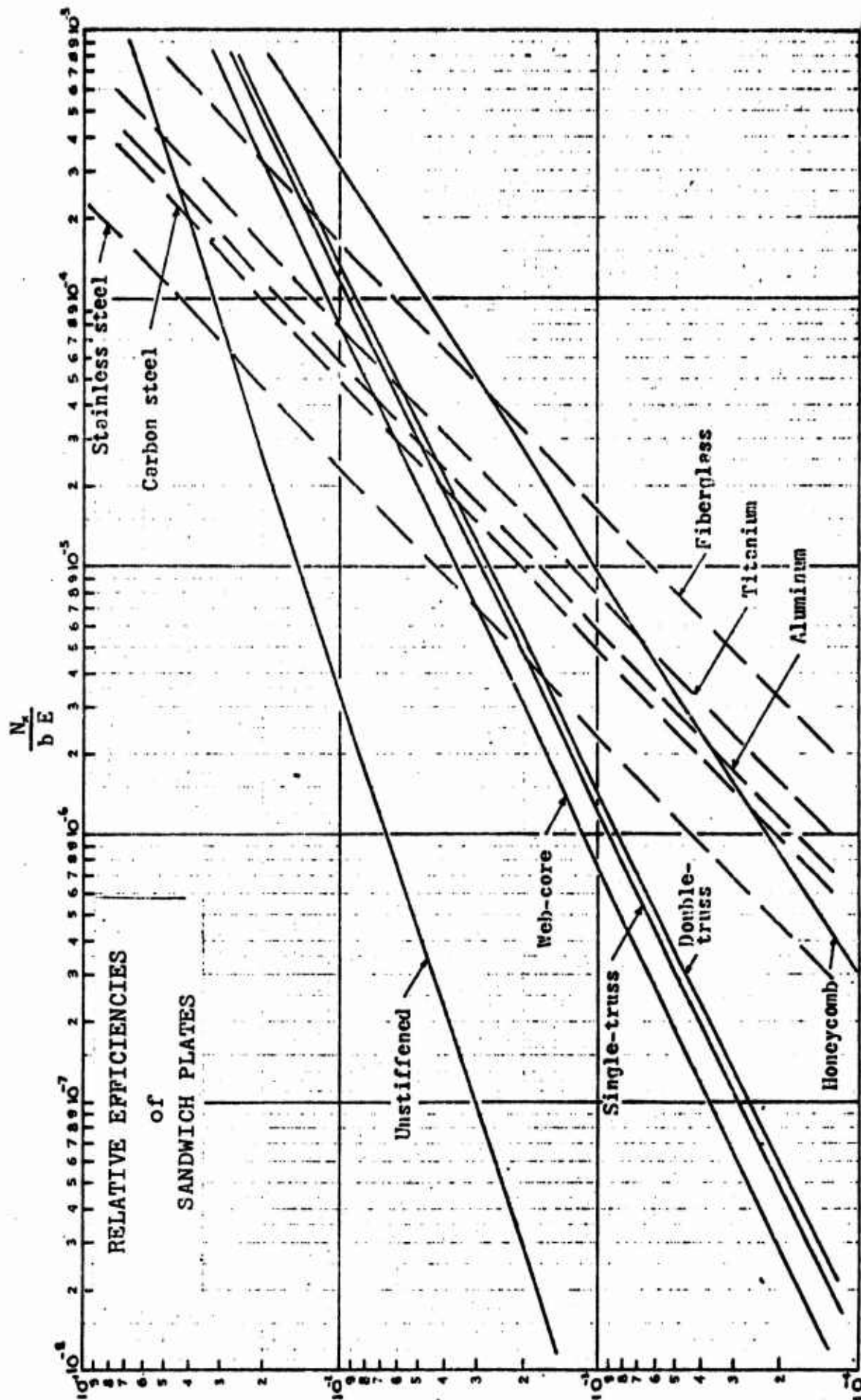


Figure 10

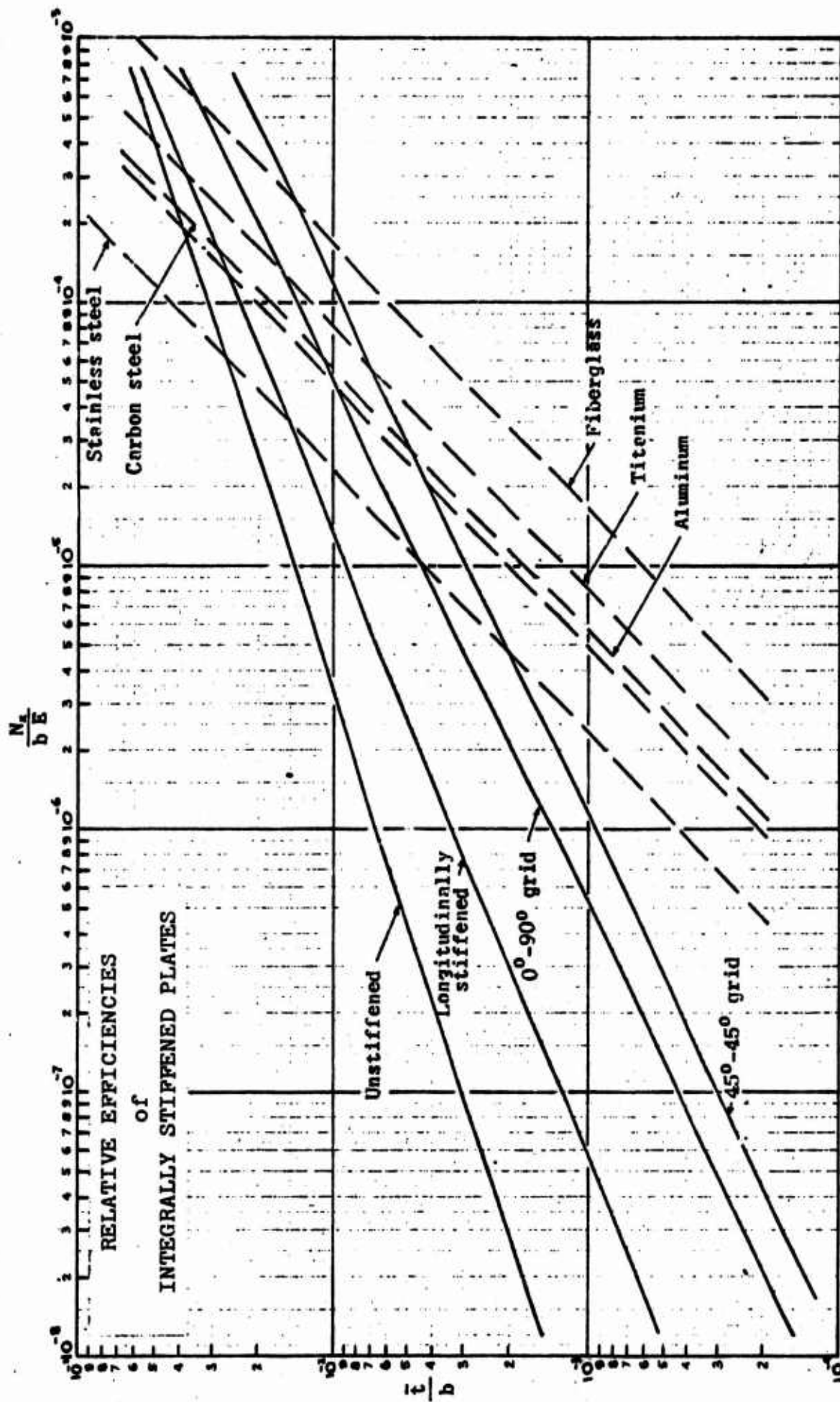


Figure 11

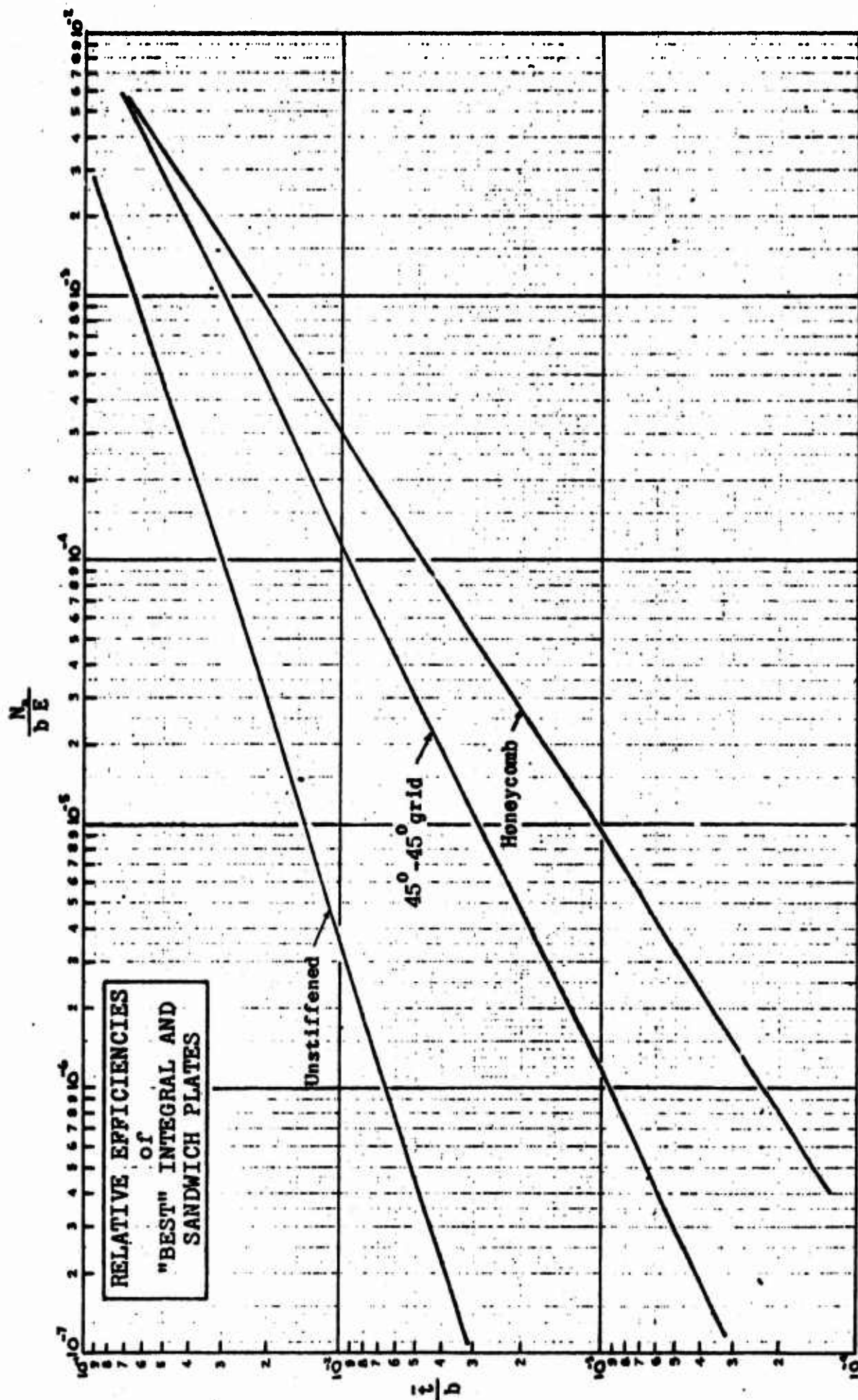


Figure 12

Johnston and Lantz (Ref. 9) state that the 0° - 90° grid is more efficient than the 45° - 45° grid because the crooked load path followed in the later configuration causes higher internal stresses which lead to yielding at lower load levels. This situation is not considered here and, thus, the 45° - 45° grid is shown to be the more efficient of the two.

COST CONSIDERATIONS

Minimum weight and minimum cost are generally identical for large structures, such as aircraft, spacecraft, and buildings. However, when one is concerned with a relatively small structural component, such as a stiffened plate, such a general conclusion may not be valid for all cases. For example, a very light (and, therefore, very small) integrally stiffened plate requires more intricate machining than a larger and heavier plate. Thus, the small plate may be more costly.

No work has been performed in the area of cost-weight optimization of stiffened plates for two possible reasons. First, a mathematical formulation of the problem appears extremely complex. Second, very little information is available on the production costs of various plate configurations.

The only available cost figures pertaining to the production of integrally and externally stiffened plates are those given by Williams (Ref. 29) for a series of 2 ft. x 10 ft. panels. From this limited amount of information, the writer has constructed a family of curves (Fig. 13) which compare the cost-weight efficiencies of three stiffened plates in axial compression:

- a. Longitudinal, flanged, integral stiffeners (Type A),
- b. Longitudinal, unflanged, integral stiffeners (Type B),

c. Longitudinal, external Z-stiffeners
(Type C).

The cost data is based upon the production rates of the British aircraft industry. Although the costs in the United States may be quite different, it can be assumed that the relative production costs of the three configurations are nearly the same. For this reason, Figure 13 provides a worthwhile guide and some insight into the cost-weight problem.

The following conclusions are reached from this graphical representation:

a. Type A: The flanged integrally stiffened plate is the most efficient of the three, from the standpoint of weight, for lower-level loadings. It is the most expensive of the three for the entire range of optimum weights. The minimum cost panel is one weighing approximately 40 lbs. As the weight (and size) decreases, the cost increases due to the increased production problems. As the weight increases from the optimum, the cost of materials increases, causing the curve to rise.

b. Type B: The unflanged integrally stiffened plate is the least efficient, from a weight standpoint, for large loads. It is, however, less costly, over the entire range, than the Type A plate.

c. Type C: The Z-stiffened plate is not the most efficient, from the standpoint of weight, for any loading

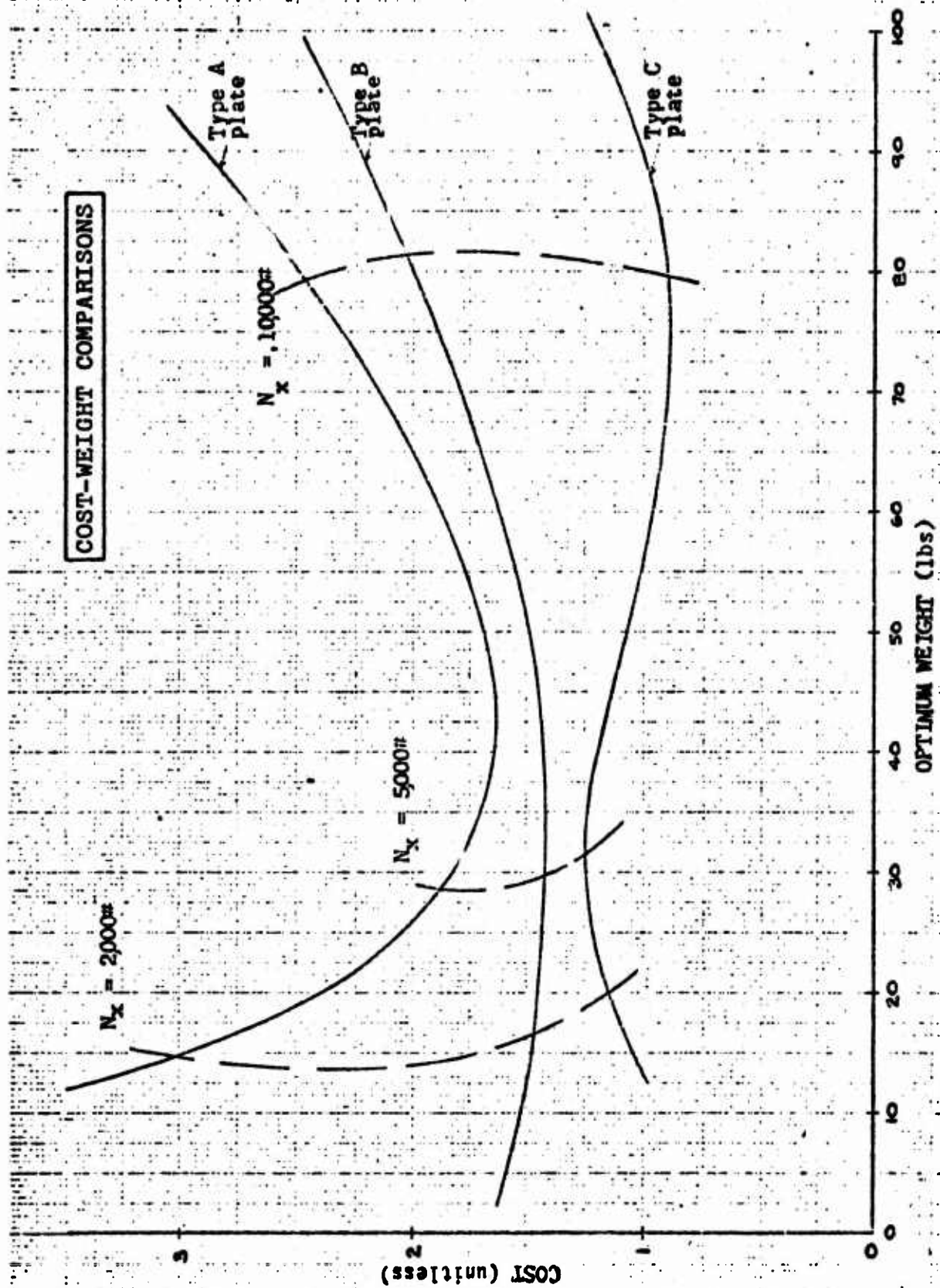


Figure 13

condition. But, it is the cheapest of the plates considered for the entire range of optimum weights.

From this comparison, one would conclude that the externally stiffened plate should be chosen over the integral panel. Before such a decision is reached, however, other factors must be considered. For example, the externally stiffened plate includes (and the integral plate does not) bolt and rivet holes which reduce the effective cross-sectional areas and act as stress raisers. The same holes are also potential leak paths in plates which are designed to aid in containing fluids.

Production-cost data is not available for sandwich construction. It appears likely, however, that minimum cost, in a sandwich plate, equals minimum weight and that a separate cost study is unnecessary.

SUMMARY

The minimum-weight analysis presented here is of a relatively crude nature. It ignores loadings other than that of direct compression, and it is based upon the assumption that all buckling-mode failures occur simultaneously. But, even with these simplifications, the comparative study is of importance to the preliminary design stage of the evolution of a stiffened plate structure. During this stage, extremely important decisions regarding general configuration and material selection must be made, and a rational minimum-weight study, even one of a relatively crude nature, aids the designer in making such decisions with a high degree of confidence.

It appears now that some important steps in the general area of optimum design, and particularly that of orthogonally stiffened plates and shells, must be taken before any greater progress toward a more rigorous technique of optimization of structural systems is realized. It appears that concrete steps toward such a goal are being taken with the development of probabilistic (reliability) models for structural optimization. Important work in the area of structural reliability has already appeared in the literature (Refs. 3, 8), but a great deal remains to be done toward the application of these concepts to the design of stiffened plates.

A systematic, controlled, test program, designed to substantiate the various optimization techniques (minimum-weight, synthesis, reliability) is needed at this time to fill a generally recognized, but otherwise ignored, void in the study of optimum structures. A great deal of such work has been performed, in this country and in Great Britain, on stiffened wide columns, but the results cannot be indiscriminately applied to the behavior of plates.

It is shown here that honeycomb-sandwich construction is superior to other forms of stiffened plates. The fallacy of such a general statement, however, is exposed with the introduction of realistic material properties into the comparison (Fig. 10). For presently available materials, honeycomb construction is practical only under conditions of very low structural indexes. Additionally, it appears probable that if data were available for the generation of a cost-comparison curve, such as Figure 13, for sandwich plates, the state of today's manufacturing technology would cause the honeycomb sandwich to drop down on a rating scale of all stiffened plates.

The role of the production engineer in the area of optimum design of stiffened plates lies in producing better and less costly structures more quickly. His most positive contribution lies in the vigorous pursuit of the development of better manufacturing techniques to meet high strength/weight requirements, the reduction of production costs, and

the reduction of manufacturing time. Additionally, the production engineer has the duty, one which has been altogether neglected, to make accurate and periodic reports of a quantitative nature in professional publications in order to provide data which is essential to the researcher who is attempting to develop the optimum structure. Without the availability of such information in the future, and without a large-scale testing program to verify results, optimum design of orthogonally stiffened plates will remain an area of merely qualitative conclusions and decisions.

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